MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 SESSION (TERM ID 2210)

PPS0016 – INTRODUCTION TO PROBABILITY AND STATISTICS

(All sections / Groups)

7 FEBRUARY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 6 pages (excluding the cover page) with 5 questions.
2. Answer all questions.
3. Unless stated otherwise, if an answer is given as a decimal, it should be rounded to four significant figures.
4. Write your answers in the Answer Booklet provided.
5. Show all relevant steps to obtain maximum marks.

Question 1 [20 marks]

The following data give the fuel economy (in kilometres per litre) of 50 cars.

9.2	6.1	7.5	13.5	7.1	9.6	11.1	5.5	8.5	9.5
15.2	7.7	8.6	5.4	7.9	5.1	8.4	8.5	18.4	6.5
10.6	17.1	8.5	10.2	9.1	8.3	11.1	11.1	7.7	9.1
9.0	7.2	9.8	11.1	6.7	13.1	10.4	12.7	14.0	9.4
12.9	5.6	8.1	9.1	6.4	13.6	10.7	10.9	7.4	11.6

(a) By using 10 classes and the range of the data, determine the class size.

[1 mark]

(b) Complete the following frequency table with 10 classes. Use the smallest value as the lower limit of the first class.

[6 marks]

Class Limit	Class Boundary	Frequency

⋮

(c) Draw a frequency histogram for the data. [8 marks]

(d) Is the histogram normal, skewed left or skewed right? [1 mark]

(e) Which histogram shape is ideal: normal, skewed left or skewed right? Justify your answer. [2 marks]

(f) What is the variable in this problem? Is the variable qualitative or quantitative? [2 marks]

Continued...

Question 2 [20 marks]

(a) According to a survey of internet browser usage, the probability that a person uses Chrome given that he uses Firefox is $\frac{1}{3}$, the probability that a person uses Firefox given that he uses Chrome is $\frac{2}{7}$, and the probability that a person uses Firefox is $\frac{9}{25}$.

(i) Find the probability that a person uses both Chrome and Firefox. [2 marks]

(ii) Find the probability that a person uses Chrome. [2 marks]

(iii) Find the probability that a person uses Firefox only. [2 marks]

(iv) Find the probability that a person uses Chrome only. [2 marks]

(v) Find the probability that a person uses neither Chrome nor Firefox. [2 marks]

(b) Darren can choose between three modes of transportation to get to work. Because of heavy traffic, taking the car means that he will be late 50% of the time. The bus is late only 20% of the time, but it is usually overcrowded. Finally, the commuter train is late only 1% of the time, but it is more expensive. He takes each mode with a relative frequency of 0.3, 0.45, and 0.25 respectively. Let C, B, and T be the events that he goes to work by car, bus, and train respectively whereas L is the event that he is late.

(i) Draw a tree diagram to represent the given information. No need to show the leaves. Use the correct notation for events. [4 marks]

(ii) Find the probability that he is not late on a particular day. [2 marks]

(iii) Given that he is late on a particular day, find the probability that he drove to work. [2 marks]

(iv) Given that he didn't take a train on a particular day, find the probability that he is late. [2 marks]

Continued...

Question 3 [20 marks]

(a) The random variable X has the following probability mass distribution.

x	1	3	5	7	9
$P(X=x)$	0.2	j	0.2	k	0.2

(i) Find the values of j and k, given that $E(X) = 4.6$. [6 marks]

(ii) Find $P(4 \le X \le 8)$. [2 marks]

(iii) Construct the cumulative distributive function $F(x)$ in tabular form. [2 marks]

(b) A random variable X has probability density function given by:

$$f(x) = \begin{cases} k, & -1 \le x < 0 \\ 3x + \frac{1}{2}, & 0 \le x \le \frac{1}{3} \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

(i) Show that the value of k is $\frac{2}{3}$. [4 marks]

(ii) Find and write the cumulative distribution function $F(x)$ as a piecewise function. [6 marks]

Continued...

Question 4 [20 marks]

(a) The number of babies born each hour at a hospital follows a Poisson distribution with a mean of 2.

(i) Find the probability that exactly five babies will be born during a particular 1-hour period at this hospital. [2 marks]

(ii) What is the expected number of babies born in a week? [2 marks]

(iii) What is the probability that a doctor delivers 14 to 16 babies during his 8-hour shift? [4 marks]

(b) 11% of the world population have type B blood. A blood drive is being held at your school.

(i) What is the probability that 2 of the first 10 blood donors have type B blood? [2 marks]

(ii) What is the probability that at most 2 of the first 10 blood donors have type B blood? [4 marks]

(iii) In how many of the 25 donors do you expect to have type B blood? [2 marks]

(c) Given that Z is a standard normal random variable. Rewrite the following expressions such that the cumulative standardized normal distribution table provided in the appendix can be used to find the corresponding probability. No need to calculate any probability value.

(i) $P(-1.80 < Z < 2.40)$ [1 mark]

(ii) $P(Z > -1.22)$ [1 mark]

(iii) $P(1.23 < Z < 1.25)$ [1 mark]

(iv) $P(Z > 1.20)$ [1 mark]

Continued...

Question 5 [20 marks]

(a) Given the population data 4, 4, 5, 6, and 6.

(i) How many samples of size 2 are there? [1 mark]

(ii) Construct the sampling distribution of the sample mean for samples of size 2 by filling in the following two tables. Use subscript to differentiate the same data values, for example, 6_1 and 6_2. [15 marks]

No	Sample	Sample mean

⋮

Sample mean	Frequency	Probability of sample mean

⋮

(iii) What shape does the population distribution have? [1 mark]

(iv) What shape does the sampling distribution of the sample mean have? [1 mark]

(b) State whether the following statement is TRUE or FALSE.

(i) If the population distribution is skewed left, then the sampling distribution of the sample mean is skewed left. [1 mark]

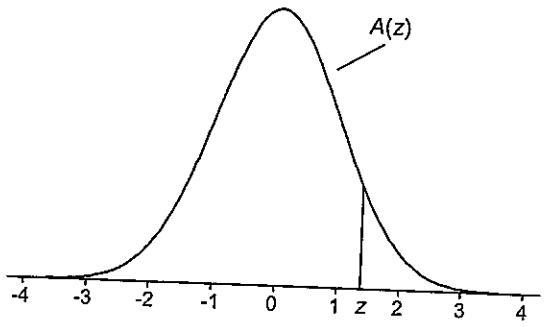
(ii) If the population distribution is normal, then the sampling distribution of the sample mean is normal. [1 mark]

Continued...

Appendix

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							